B.Sc II Paper III

Corollary of Archimedean property of Real numbers.

Corollary 1. If a be a positive real number and b any real number then their exists a positive integer n such that $na > b$.

Corollary 2. For any positive real number a there exists a positive integer n such that $n > a$
The result follows by considering the two positive real numbers 1 and a.

Corollary 3 - For any $\varepsilon > 0$ there exists a positive integer n such that $1/n < \varepsilon$.

Corollary 4. If a be any real number then there exists a positive integer n such $n > a$. For $a \leq 0$, any positive integer $n > a$ and for $a > 0$

Dedekind's theory of real numbers

Dedekind's Property –

If L and U are two subsets of R such that

(i.) $L = \varphi$, $U = \varphi$ (each class has at least one member)

(ii.) $L \cup U = R$ (Every real number has a class.

(iii.) Every member of L is less than every member of U i.e

$$x \in L \wedge y \in U \Rightarrow x < y.$$

then either L has the greatest member or U has the smallest number.

Sequences — A function whose domain is the set N of natural numbers and Range a set of real numbers is called a real sequence.

Thus real sequence is denoted by symbolically as $S : N \to R$

Notation Since the domain for a sequence is always N, a sequence is specified by the values S_n, $n \in N$. Thus a sequence may be denoted as

$$\{S_n\},\ n \in N \text{ or } \{S_1, S_2, S_3, \ldots S_n \ldots\}$$

The values of $S_1, S_2, S_3 \ldots$ are called the first, second .... terms of the sequence.

The mth and nth terms S_m and S_n for $m \neq n$ are treated as distinct terms even if $S_m = S_n$

The number of terms in a Sequence is always infinite.

In Other words,

A Sequence as an ordered set of real numbers can be put in a one-one correspondence with the set of natural number. However a sequence may have only a finite number of distinct elements.

For example

1. $\{S_n\} = \{(-1)^n\}, n \in N$

Here $S_1 = -1$ $S_2 = 1$, $S_3 = -1$, $S_4 = 1 \ldots$ so that there are only two $1, -1$ distinct elements.

2. $\{S_n\} = \left\{\frac{1}{n}\right\} n \in N$

Here $S_1 = 1$ $S_2 = \frac{1}{2}$ $S_3 = \frac{1}{3}$ ____

All elements are distinct.

The Range

The range or Range set is the set consisting of all distinct elements of a sequence, without repetition and without regard to the position of a term. Thus the range may be a finite or a infinite set, without ever being the null set.

Bounds of a Sequence

Bounded above sequence

A sequence $\{S_n\}$ is said to be bounded above if there exists a real number K such that

$$S_n \leq K \quad \forall \; n \in N$$

Bounded below Sequences – A sequence $\{S_n\}$ is said to be bounded below if their exists a real number

Page No.
Date: / /

k such that

$$S_n \geq k \quad \forall\, n \in N$$

Bounded Sequence

A sequence is said to be bounded when it is bounded both above and below. K and k are respectively the upper and the lower bounds of the sequence.

Note A sequence is bounded iff its range is bounded

Also the bounds of the range are the bounds of the sequence.